## **Model of the optical Stark effect in semiconductor quantum wells: Evidence for asymmetric dressed exciton bands**

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The influence of intense coherent  $\omega$  and  $2\omega$  laser beams on the electric properties of quantum wells is investigated. In the optical quantum-interference process, an asymmetric dressed band structure can be achieved in *k* space. By adjusting the relative phase and the polarization direction of the  $\omega$  and  $2\omega$  laser beams, the light-induced shift and group velocity variation in electrons and spins can be tuned. The transport effects of asymmetric dressed carriers are studied. We find that if the pseudospin Hall conductance (p-SHC) is dominated by the optically induced band mixing, the p-SHC is nearly invariable with the relative phase of the laser beams. But if the p-SHC is caused by the disorder scattering effect, it is sensitive to the relative phase of the laser beams.

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The study of band structure in solid is of fundamental interest in basic physics as well as in material science. In traditional materials with time-reversal symmetry and spaceinversion symmetry, the electron states are symmetric in *k* space. Can we find an asymmetric band structure in *k* space? As shown in the research of photonic crystals, if timereversal and space-inversion symmetries are broken, the bands are asymmetric with respect to the wave vectors.<sup>1[,2](#page-3-2)</sup> In a quantum well (QW), the symmetry can be broken by applying external fields, e.g., magnetic field, electric field, and optical field.

Compared with the electric or magnetic field, the optical field control on carrier transport offers several advantages. Optical fields can control not only the charge carriers but also the spin carriers, especially which can be performed over femtosecond time scale, thanks to the advances in ultrafast laser technology. One of the fundamental methods for the optical control is based on the optical Stark effect  $(OSE).<sup>3-6</sup>$  $(OSE).<sup>3-6</sup>$  $(OSE).<sup>3-6</sup>$  The OSE in semiconductors is due to a dynamical coupling of excitonic states by an intense laser field, which have many fascinating applications, such as the generation and manipulation of the entanglement of the spin, $\frac{7}{1}$  the control of single exciton polarizations, $\delta$  photoinduced ferromagnetism, $9$  and the optical control of topological quantum transport[.10](#page-3-8) But the traditional OSE can not be used to realize asymmetric band structure in *k* space because the light-induced shifts are the same at **k** and −**k**. Therefore, an asymmetric optical transition process in *k* space is essential. The optical quantum-interference process (QUIP) between one- and two-photon absorption is such a process, which has been used to inject charge current and spin current or to detect the spin current. $11-15$  $11-15$ 

In this Brief Report, we studied the OSE of dressed bands in a semiconductor QW in the optical QUIP. In the QUIP between one- and two-photon absorptions, the transition rate depends on the relative phase and the polarization direction of the  $\omega$  and  $2\omega$  laser beams. The optical transition of QUIP can be enhanced at +**k** but vanishes at −**k**, or vice versa. Therefore, the light-induced shift of the energy levels is increased at +**k** but decreased at −**k**. The symmetry of the energy with respect to the wave vectors is broken. Then, the absolute values of the group velocities of forward and backward carriers are different. Optically controlled one-way isolator can be achieved. Because of the sub femtosecond time scale of the phase variation in a photon, the quantum systems can be easily controlled over the femtosecond time scale. Furthermore, different from the real transition processes,  $11-14$ the dressed bands for the OSE are generated only virtually in the nonresonant case; the absorption of photons is quite small and a low-power consumption is expected.

The OSE in optical QUIP can also be used to control the transport of spins. When the  $\omega$  light field is polarized along the *x* direction and the  $2\omega$  light field is polarized along the *y* direction, the transition from the hole state  $|3/2,3/2\rangle$  $(|3/2,-3/2\rangle)$  to the conduction state  $|1/2,1/2\rangle$   $(|1/2,1/2\rangle)$ −1/2) would be enhanced (weakened) at +**k** but weakened (enhanced) at −**k**. Therefore, the light-induced shift of the spin-up and spin-down states are different at +**k** and −**k**, a large spin splitting can be found. The spin splitting and the group velocities of spins can also be tuned by changing the relative phase of the  $\omega$  and  $2\omega$  laser beams.

Except for the phase controlled diodes, the asymmetric band structures in *k* space also induce other fascinating transport effects. We study the Berry curvature and p-SHC of dressed bands in optical QUIP as an example. The Berry curvature can be analogous to a magnetic field in the crystal momentum space, which have been used in the calculation of the anomalous Hall effect, the spin Hall effect, and Bose-Einstein condensates. $10,16$  $10,16$  In optical OUIP, an asymmetric Berry curvature can be found, and it can be tuned by the relative phase of the  $\omega$  and  $2\omega$  laser beams.

We first explain the two-color laser-induced band renormalization for the case of noninteracting electrons and then consider the influence of the Coulomb interactions within the Hartree-Fock (HF) approximation. In the case of nonresonant excitation, the renormalized electron spectrum within a rotating-wave approximation is<sup>4</sup>

$$
E_e^0(\mathbf{k}) = \frac{1}{2} \{ \varepsilon_{1\mathbf{k}}^0 + \varepsilon_{2\mathbf{k}}^0 + 2\omega + [(\varepsilon_{1\mathbf{k}}^0 - \varepsilon_{2\mathbf{k}}^0)^2 + 4|\Delta_{\mathbf{k}}^0|^2]^{1/2} \},
$$
\n(1)

where  $\epsilon_{1k}^{0}(\epsilon_{2k}^{0})$  is the unperturbed conduction- (valence-) band energy,  $\Delta_{\mathbf{k}}^0 = \mu(\mathbf{k}) \Xi_p$  is the Rabi frequency with zero detuning,  $\Xi_p$  the electric field amplitude,  $\mu(\mathbf{k})$  the interband dipole matrix element, which depend on not only the value of the wave vector but also on the direction of the wave vector. Instead of perturbation theory, we calculate the interband dipole matrix element for the one- and two-photon absorptions using a Volkov-type wave function, which describes the electron state in the presence of an optical field of arbitrary intensity. $15,17,18$  $15,17,18$  $15,17,18$  In optical QUIP, the total vector potential of  $\omega$  and  $2\omega$  laser beams is given by **A**  $= \mathbf{a}_1 A_1 \cos(\omega t + \varphi_1) + \mathbf{a}_2 A_2 \cos(2\omega t + \varphi_2)$ . Thus,  $|\Delta_k^0|^2$  for a single- and two-photon process can be written as

$$
|\Delta_{\mathbf{k}}^{0}|^{2} = \frac{2\hbar\omega}{m_{0}E_{s}^{0}e_{0}}(T_{\mathbf{k}}^{n} + T_{\mathbf{k}}^{i}),
$$
 (2)

<span id="page-1-0"></span>where

$$
T_{\mathbf{k}}^{n} = \left(\frac{\eta_{1}\mathbf{k} \cdot \mathbf{a}_{1}}{2}\right)^{2} |\mathbf{p}_{vc} \cdot \mathbf{a}_{1}|^{2} A_{1}^{2} + |\mathbf{p}_{vc} \cdot \mathbf{a}_{2}|^{2} A_{2}^{2},
$$
  

$$
T_{\mathbf{k}}^{i} = A_{1} A_{2} \frac{\eta_{1}\mathbf{k} \cdot \mathbf{a}_{1}}{2} \{ (\mathbf{p}_{vc} \cdot \mathbf{a}_{1})^{*} (\mathbf{p}_{vc} \cdot \mathbf{a}_{2}) e^{i(2\varphi_{1} - \varphi_{2})} + \text{c.c.} \},
$$

where  $m_0$  is the electron mass,  $E_s^0$  the exciton transition energy,  $e_0$  is the electron charge magnitude,  $\eta_1 = \frac{e_0 A_1}{\omega m_{cv}}$ ,  $1/m_{cv}$  $= 1/m_c - 1/m_v$ , and  $m_c(m_v)$  the effective mass of electrons (holes),  $\mathbf{p}_{vc} = \langle c | \mathbf{p} | v \rangle$  the dipole transition matrix element, due to the quantum-confinement effect in QW potential, the hole state  $|v\rangle = \sum_{j=1}^{4} g^{j}(\mathbf{k}) U^{j}$ , where  $U^{j}$  is the Bloch state for spin component  $j = (\frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2})$ ,  $g^j(\mathbf{k})$  corresponds to the mixing of the heavy- and light-hole states, which are calculated using the effective mass theory based on the  $4 \times 4$  Luttinger valence-band Hamiltonian. Thus for a wide QW, the dipole transition matrix element  $\mathbf{p}_{v\epsilon}$  in Eq. ([2](#page-1-0)) reduces with increase the QW width.<sup>15[,19](#page-3-16)</sup>  $T_k^n$  and  $T_k^i$  describes the photon absorption (single-photon and two-photon) and the quantum interference between the one- and two-photon excitations, respectively. The Rabi frequency can be strongly asymmetric at  $\pm \mathbf{k}$ , e.g.,  $|\Delta^0_{\mathbf{k}}|^2 \neq 0$ , while  $|\Delta^0_{-\mathbf{k}}|^2 = 0$  by adjusting the polarization and the relative phase of the pulses.

<span id="page-1-1"></span>When the Coulomb interactions are included within the HF approximation, the renormalized band energies and the Rabi frequency are given by $4,10$  $4,10$ 

$$
\varepsilon_{i\mathbf{k}} = \varepsilon_{i\mathbf{k}}^0 + 2V_{\mathbf{q}=0} \sum_{j,\mathbf{k}'} n_{j\mathbf{k}'} - \sum_{j,\mathbf{k}'} V_{\mathbf{k},\mathbf{k}'} n_{j\mathbf{k}'},\tag{3}
$$

$$
\Delta_{\mathbf{k}} = \Delta_{\mathbf{k}}^0 + \sum_{\mathbf{k}'} V_{\mathbf{k},\mathbf{k}'} \psi_{\mathbf{k}'},
$$
\n(4)

<span id="page-1-2"></span>where  $V_{\mathbf{k},\mathbf{k}'}=V_{\mathbf{q}=\mathbf{k}-\mathbf{k}'}$  is the Coulomb potential,  $n_{\mathbf{j}\mathbf{k}'}$  is the conduction- and valence-distribution function, and  $\psi_{\mathbf{k}}$  is the polarization induced by the pump field. The Coulomb interaction couples transition with different **k** so that  $\Delta_{-\mathbf{k}}$  is not

<span id="page-1-3"></span>

FIG. 1. (Color online) The contour plot of (a) the dispersion relation and (b) the group velocities of charge carriers as a function of the wave vector **k** at  $2\varphi_1 - \varphi_2 = 0$  with parallel linearly polarized laser beams. (c) The band energy and (d) the group velocities of charge carriers as a function of relative phase of the  $\omega$  and  $2\omega$  laser beams at different wave vectors,  $k_x$ =+0.03 nm<sup>-1</sup> (black solid line),  $k_x$ =−0.03 nm<sup>-1</sup> (red dashed line),  $k_x$ =+0.015 nm<sup>-1</sup> (green dashdotted line), and  $k_x = -0.015$  nm<sup>-1</sup> (blue dotted line).

zero even if the transition vanished at −**k**. Thus strong Coulomb interactions will reduce the asymmetry of the band structure in *k* space. Since the Coulomb potential is inversely proportion to **q**, the coupling between +**k** and −**k** is small for the large  $q(=2k)$ . And for large pump intensities, the Coulomb interactions only introduces a weak coupling compared to the external optical field. Therefore, even with the Coulomb interactions, a strong asymmetric band structure of dressed states can be seen.

Numerical calculations of Eqs.  $(3)$  $(3)$  $(3)$  and  $(4)$  $(4)$  $(4)$  by the HF approximation are performed for a  $GaAs/Al<sub>0.35</sub>Ga<sub>0.65</sub>As QW$ with a thickness of  $d=10$  nm and a symmetric confinement in the growth direction. The pump detuning  $E_s^0 - 2\omega$ = 5 meV, and the pump intensity of the  $\omega$  and  $2\omega$  laser spot is  $I_{\omega}$ = 100 GW cm<sup>-2</sup> and  $I_{2\omega}$ = 50 MW cm<sup>-2</sup>, respectively, which is not difficult to implement using current ultrafast laser techniques. And in a microcavity, a large Rabi frequency can be achieved even with a relatively weak laser field.

Figure  $1(a)$  $1(a)$  shows dispersion relations of the dressed bands in optical QUIP with parallel linearly polarized  $\omega$  and  $2\omega$  laser beams. Since the optical transition of QUIP is enhanced at +**k** but reduced at −**k**, the light-induced blueshift at +**k** is larger than that at −**k**. The wave vector corresponding to the minimum of dressed band is about  $-0.03$  nm<sup>-1</sup> and the symmetry of the band structure in *k* space is broken. As shown in Fig.  $1(c)$  $1(c)$ , the asymmetry can be tuned by changing the relative phase of the  $\omega$  and 2 $\omega$  laser beams. At  $2\varphi_1-\varphi_2$  $= 0$ , the energy of the dressed band at  $k=+0.03$  nm<sup>-1</sup> is about 7 meV, and at  $k=-0.03$  nm<sup>-1</sup> the energy of dressed band is about 4.5 meV. Thus, the splitting is about 3.5 meV, which can be detected experimentally.

<span id="page-2-0"></span>

FIG. 2. (Color online) (a) The contour plot of the spin splitting and (b) the group velocities difference of spin-up and spin-down carriers as a function of the wave vector **k** with cross-linearly polarized laser beams. (c) Spin splitting as a function of the relative phase of the  $\omega$  and  $2\omega$  laser beams at different wave vectors. (d) The group velocities of spin-up and spin-down carriers at *k*= +0.035 nm<sup>-1</sup> (upper part) and  $k=-0.035$  nm<sup>-1</sup> (lower part) as a function of relative phase of the  $\omega$  and  $2\omega$  laser beams.

As a result of the asymmetric band in *k* space, the absolute values of group velocities of forward and backward charge carriers are different. As shown in Fig.  $1(b)$  $1(b)$ , the group velocity of carriers at +**k** is much large than that at −**k**. And by varying the relative phase of the  $\omega$  and  $2\omega$  laser beams [see Fig.  $1(d)$  $1(d)$ ], some interesting results can be achieved, for instance, at  $2\varphi_1 - \varphi_2 \approx 30^\circ$ , the group velocities of the charge carriers at  $k=-0.03$  nm<sup>-1</sup> can be tuned to nearly zero, but the charge carriers at  $k=+0.03$  nm<sup>-1</sup> still have very high group velocities, which may lead to important technological applications, such as the phase controlled one-way isolator.

The OSE in optical QUIP with circular polarized or crosslinearly polarized laser beams can also be used to control the transport of spins. In the case of cross-linearly polarized laser beams, the light-induced shift of the spin-down states at +**k** is larger than that at −**k**, and vice versa for the light-induced shift of the spin-down states. Therefore, a large spin splitting can be found, as shown in Fig.  $2(a)$  $2(a)$ . By varying the relative phase of the  $\omega$  and  $2\omega$  laser beams, the spin splitting at *k*  $= 0.04$  nm<sup>-1</sup> can also be tuned from 3 to -3 meV [see Fig.  $2(c)$  $2(c)$ ].

The large spin splitting will also induce a strongly asymmetric transport of the dressed spins. Figure  $2(b)$  $2(b)$  shows the contour plot of the group velocity differences of spin-up and spin-down carriers as a function of the wave vector **k**. The group velocity of spin-up carriers is much large than that of spin-down carriers at +**k** but vice versa for the carriers at −**k**. Furthermore, the group velocities of spins can also be tuned by the relative phase of the  $\omega$  and  $2\omega$  laser beams, e.g., at *k*=+0.035 nm−1, the group velocities of spin-down carriers is nearly zero with  $2\varphi_1 - \varphi_2 \approx 70^\circ$ , but the group velocity of spin-up carriers is about  $1 \times 10^5$  m/s [see the upper part of Fig.  $2(d)$  $2(d)$ ]. Thus, the OSE in optical QUIP with cross-linearly

<span id="page-2-1"></span>

FIG. 3. (Color online) (a) The Berry curvature distribution of the top dressed band with parallel linearly polarized laser beams. (b) p-SHC as a function of the relative phase of the  $\omega$  and  $2\omega$  laser beams. Red dashed, blue dash-dotted, and black solid lines denote the clean contribution, the side-jump contribution, and the total Hall conductance, respectively.

polarized laser beams can be used in spin diodes or one-way isolators. And it possesses several advantages compared with the spin field-effect transistor control by a gate voltage, $^{20}$ e.g., for OSE in optical QUIP, the Rashba spin-orbit interaction (SOI) and the Dresselhaus SOI are unnecessary; thus, more semiconductor materials can be used to fabricate spintronics devices. In addition, ultrafast switching times and a low-power consumption are expected in this phasecontrolled spin diode.

The asymmetric band structure in *k* space is also potentially valuable in areas of quantum computation. In this Brief Report, we investigate the optical control topological quantum transport as an example. The *k* space Berry curvature is defined as  $\mathbf{F}^c(\mathbf{k}) = \nabla_{\mathbf{k}} \times \mathbf{R}^c(\mathbf{k})$ , where  $\mathbf{R}^c(\mathbf{k})$  $\vec{b} = i \langle \Phi_0 | \vec{b}_{c,k} e^{i\mathbf{k} \cdot \mathbf{r}} (\partial/\partial \mathbf{k}) \vec{b}_{c,k}^{\dagger} e^{-i\mathbf{k} \cdot \mathbf{r}} | \Phi_0 \rangle$ ,  $\Phi_0$  is the complete filling of all bands and  $\tilde{b}_{c,k}$  the hole creation operator for dressed states. Because of the strong spin-orbit coupling in the valence band, the dressed band has nonzero Berry curvature even without Rashba SOI and Dresselhaus SOI. The large *k* space Berry curvature has an important topological contribution to the anomalous Hall effect, where the clean contribution to the carrier Hall conductance is given by  $\sigma^{cl} = \frac{e^2}{\hbar} \sum_{\mathbf{k}} [1 - f(\mathbf{k})] F_z^c(\mathbf{k})$ . The disorder effects will lead to a side-jump contribution to the Hall conductance  $\sigma^{sj}$  $=-\frac{e^2}{\hbar} \sum_{\mathbf{k}} F_z^c(\mathbf{k}) \frac{\partial f(\mathbf{k})}{\partial \mathbf{k}} \cdot \mathbf{k}$ .<sup>[10](#page-3-8)</sup> The skew scattering is not included since in this lightly doped semiconductors the optically induced Hall conductance is much stronger than that from skew scattering.<sup>21</sup>

Figure  $3(a)$  $3(a)$  shows the Berry curvature distribution of the dressed band with a parallel linearly polarized laser pump. When the  $\omega$  light field and the  $2\omega$  light field are parallel linearly polarized along the *x* direction, the optical transition rates of the QUIP at +**k** and −**k** are different. Consequently, an asymmetric Berry curvature occurs. But the light-induced blueshift of the spin-up and spin-down bands have equal magnitudes. Thus, the Hall conductance of the spin-up and spin-down bands,  $\sigma_{\uparrow}$  and  $\sigma_{\downarrow}$ , have equal magnitude but opposite sign,  $\sigma_{\uparrow} = -\sigma_{\downarrow}$ . In this system, a p-SHC can be defined as  $\sigma_s = \sigma_\uparrow - \sigma_\downarrow$ .<sup>[10](#page-3-8)</sup>

Figure  $3(b)$  $3(b)$  shows the p-SHC as a function of the relative phase of the laser beams. In one- and two-photon absorption, the interference term  $T_k^i$  only changes the distribution of the transition rate in *k* space; the total transition rate is invariant with respect to the relative phase of the  $\omega$  and  $2\omega$  laser beams. Since the clean contribution to the p-SHC  $\sigma^{cl}$  is the sum of the Berry curvature in *k* space, the interference between one- and two-photon absorptions has little influence on  $\sigma^{cl}$  [see the red dashed line in Fig. [3](#page-2-1)(b)]. But the case is different for the p-SHC caused by the side-jump contribution. The side-jump process, as does the one- and two-photon absorption in QUIP, depends not only on the value of the wave vector but also the direction of the wave vector. The p-SHC caused by the side-jump contribution  $\sigma^{sj}$  is more sensitive to the relative phase of the laser beams [see the blue dash-dotted line in Fig.  $3(b)$  $3(b)$ ]. Thus, if p-SHC is dominated

by the optically induced band mixing, the p-SHC is nearly invariable with the relative phase of the laser beams. However, if the p-SHC is caused by the **k**-depended scattering processes, it strongly depends on the relative phase of the laser beams. This feature is helpful for distinguishing whether the spin-Hall conductance is induced by the spinorbit coupling or by scattering processes. $21-25$  $21-25$ 

In summary, the OSE of excitons under intense coherent  $\omega$  and  $2\omega$  laser beams is investigated. Asymmetric band structures of the dressed states are achieved. By adjusting the relative phase and the polarization direction of the  $\omega$  and  $2\omega$ laser beams, the light-induced shift and the group velocities of forward and backward carriers can be tuned, which is important in the fabrication of the phase-controlled electron or spin diodes with ultrafast switching times and low-power consumption. The transport effects of asymmetric dressed carriers are studied. The p-SHC is nearly invariable with respect to the relative phase of the laser beams if the optically induced band mixing dominates. But the p-SHC due to the **k**-depended scattering processes strongly depends on the relative phase of the laser beams.

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